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An alternative approach to the Busk construction for a single surface

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Abstract

An alternative, exact construction is given, both in geometrical form and in terms of co-ordinate geometry suitable for computation, for the problem of linking two known points of given dips on a single surface by two circular arcs, mutually tangent at their point of contact and tangent to the surface at the given points. The problem was originally analysed by Busk in 1929.

1. Introduction

The 'Busk construction' (Busk, 1929, pp. 26–28; it is, in fact, one of many described by him) is referred to, if not wholly uncritically (see e.g. Suppe, 1985), in many textbooks of structural geology. It describes the construction, given limited dip information, of the profile of a folded bedding surface as a series of circular arcs, and assumes cylindrical folding to extrapolate between surfaces.

Two occurrences of the same surface cannot, in general, be linked by a single circular arc, but they can by two arcs which have a common tangent plane at their intersection. In fact, as Busk realised, a range of such solutions is possible. He offered two graphical solutions, one approximate (repeated by Wojtal, 1988) and one accurate (repeated by Higgins, 1962), for constructing the arcs. The purpose of this note is to present an accurate graphical solution alternative to that of Busk, and to explore more fully than he did the range of possible solutions.

It transpires that the points of common tangency of the two arcs lie on yet another circular arc, and the range of possible two-arc solutions is bounded by the two single arcs which are respectively tangent to bedding at one of the points and passing through the other. As limiting cases of two-arc solutions, these can be thought of as combining an arc of finite radius and an arc of zero radius, the latter to accommodate the angular divergence between the arc and bedding at the second point. The range of possible solutions is a function of the relative positions of the two points and the dips at the points: see Fig. 1 for examples. As usual, unambiguous interpretations require close control. The two arcs may turn in the same sense or in opposite senses; where, as in Fig. 1(a), both possibilities occur together, the two classes of solution are separated by the special case in which one arc becomes a straight line (cf. Higgins, 1962).

In a geometrical sense, the construction described here is less pure than that of Busk, in that it requires the use of a protractor as well as, rather than only, ruler and compasses. The construction, however, is straightforward, and lends itself readily to computed plotting.

2. Graphical construction

The required surface is known at the two points A and B (Fig. 2a), at which the dips are, respectively, δ_1 and δ_2'. Since a number of different bedding dip combinations arise it is necessary to define a consistent angular convention. Bedding dips at A and B are taken as

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positive if the bed is right-way-up and dips down to the right. In geometrical arguments it is sometimes easier to work with angles that, on this convention, are negative: these are denoted by dashes, e.g. $\delta_2' = -\delta_2$.

First map the locus of the point of common tangency. Then, for a chosen point on this locus, the two arcs can be constructed:

1. To construct the arc which is the locus of the point of common tangency, draw a line through A at an angle to AB of:

$$\sigma = (\delta_1 - \delta_2)/2 - 90 \tag{1}$$

where σ is measured positive clockwise from AB (and is negative for the case of Fig. 2a). Extend it as required to cut the perpendicular bisector of AB at K, the arc centre; the arc radius is KA: see Fig. 2(a).

2. To construct a possible two-arc solution, select a point O on the previous arc. Draw the common tangent at O: although the point O lies on a circular arc, the common tangent sought is *not* tangent to this arc. Rather, the tangent makes an angle with OA, labelled α in Fig. 2(b), which is equal to the angle between the bedding at A and the line OA; the two angles form the base angles of the isosceles triangle AMO. (One could, equivalently, work from the point B using the angles labelled β' in Fig. 2b.) Draw perpendiculars to the bedding at A and B, and to the common tangent, i.e. bedding, at O. The arc centres P, for arc OA, and Q, for arc OB, are found as the intersections of perpendiculars to bedding at A and O, O and B respectively; radii are AP, BQ: see Fig. 2(b).
3. Concentric bedding horizons can be stepped off by drawing arcs on P, Q as centres and increasing or decreasing the radii by the required bed thicknesses (Fig. 2c).
4. It is not necessary, but may be of interest, to construct the limiting cases which form the envelope to the set of two-arc constructions. These are given by the pair of arcs which are respectively tangential to bedding at one point and pass through the other. The arc centres are found by the intersections, R and S, of the bedding normals at A and B with the perpendicular bisector of AB; radii are AR and BS: see Fig. 2(d).

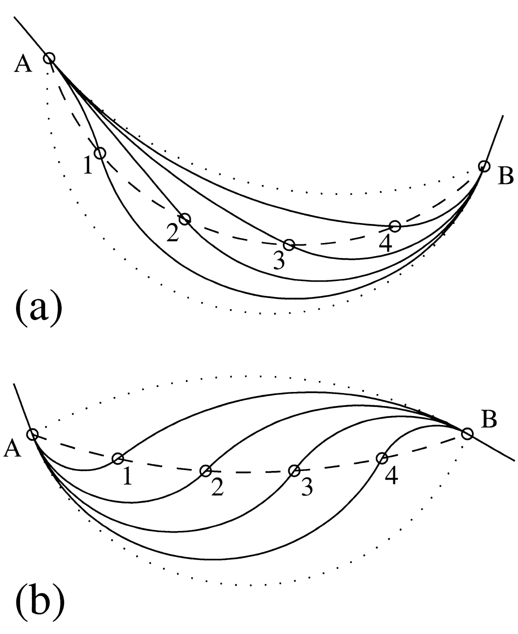

Fig. 1. Examples of the Busk construction linking two points, A and B, of given dips, on a single surface, by two mutually tangential arcs. In (a) the two arcs have (predominantly) the same senses of turn; in (b) they are opposed. The dotted arcs define the envelope of all possible solutions. The dashed arc is the locus of the point of contact, or point of common tangency, of the two constructed arcs. Points marked 1–4 are points of contact for four alternative solutions.

3. Proof

3.1. Construction of the two arcs

Cases arise where the two arcs have the same, and opposite, senses of turn: they are illustrated in Fig. 3(a) and (b). The two diagrams are labelled comparably. The chords AO and OB subtended by the two arcs make angles α and β, measured positive clockwise from the chords, with respect to the bedding at A and B. The proof may be followed on either Fig. 3(a) or (b).

At two points, A and B, the dips, δ_1 and δ_2, are known. Lines at angles α from the bedding at A and, likewise, β from B meet at O, which is the point at which the two required arcs meet. The line MN is drawn through O to define two isosceles triangles AMO and BNO. MN is parallel to the bedding at O and thus the intersections of the perpendicular to MN through O with the perpendiculars to bedding at A and B define two isosceles triangles, OAP and OBQ. Thus P and Q are the centres of two arcs, of radii OP and OQ, which are mutually tangent at O.

The angles marked α, β', 2α and $2\beta'$ may readily be established. If the lines parallel to bedding at A and B are projected to meet at X, it is clear that:

$$2\alpha + 2\beta' = \delta_1 + \delta_2', \tag{2}$$

or:

$$2(\alpha - \beta) = \delta_1 - \delta_2. \tag{3}$$

This equation establishes the link between α and β, through the bedding dips. It could, if required, be used to construct the arcs directly (i.e. without, as above, first constructing the arc of common tangency): the point O lies at the intersection of lines drawn at α from A and β (found from Eq. 3) from B.

3.2. Arc of common tangency

A circle can always be fitted to three points. So A, O, B lie on a circle, centre K: see Fig. 4. (Fig. 4 repeats the base data of Fig. 3a). We need a means of locating K.

Since the triangles AKO and BKO are both isosceles:

$$\mathrm{A\hat{O}K} = (180 - \mathrm{A\hat{K}O})/2, \tag{4}$$

$$\mathrm{B\hat{O}K} = (180 - \mathrm{B\hat{K}O})/2. \tag{5}$$

Summing:

$$\mathrm{A\hat{O}B} = 180 - \mathrm{A\hat{K}B}/2. \tag{6}$$

But, since MON is a straight line:

$$\mathrm{A\hat{O}B} + \alpha + \beta' = 180, \tag{7}$$

so:

$$\mathrm{A\hat{K}B} = 2(\alpha + \beta') \tag{8}$$

or:

$$\mathrm{A\hat{K}B} = \delta_1 + \delta_2' = \delta_1 - \delta_2 \tag{9}$$

(from Eqs. (2) and (3)).

Since the triangle AKB is also isosceles:

$$\sigma' = (180 - \mathrm{A\hat{K}B})/2 \tag{10}$$

$$\sigma = (\delta_1 - \delta_2)/2 - 90. \tag{11}$$

Since A and B are fixed points, the position of K is also fixed and independent of α. Thus, as α varies, the locus of O traces out a circular arc, centre K.

4. Computable relationships

It is useful, particularly if the construction is to be programmed, to establish the key values in terms of co-ordinate geometry. Define an axis frame with x increasing to the right, z increasing down. The points A and B are, respectively, (x_1,z_1) and (x_2,z_2); the angle α is taken as the parameter defining the required point of common tangency, O, and β is available from Eq. (3).

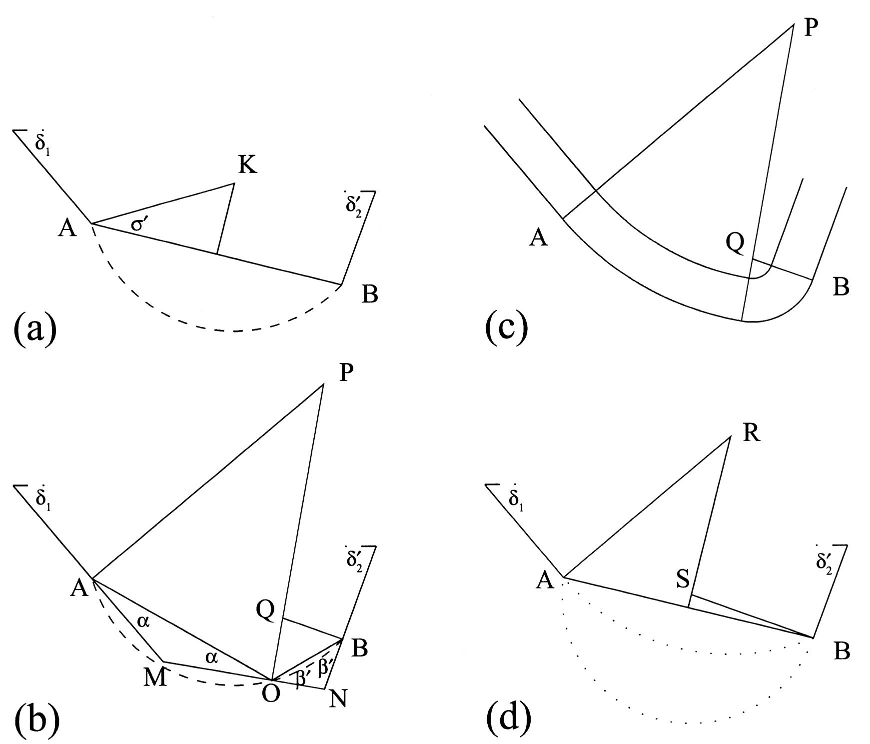

Fig. 2. (a) Construction of the locus of the point of common tangency, where the two arcs join. (b) Construction of MN, the common tangent at O, and determination of the arc centres, P and Q, and arc radii, AP and QB. (c) Bed construction. (d) Construction for limiting cases.

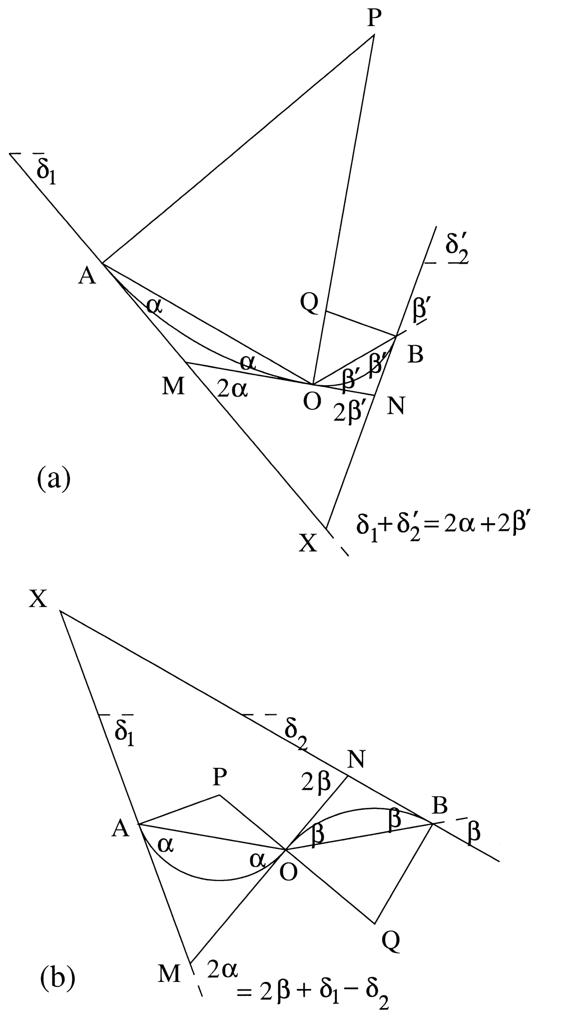

Fig. 3. Proof of the two-arc construction: (a) arcs with the same sense of turn; (b) arcs with opposite senses of turn.

The co-ordinates of O are:

$$x_0 = [z_2 - z_1 - x_2 \tan(\delta_2 - \beta) + x_1 \tan(\delta_1 - \alpha)]/[\tan(\delta_1 - \alpha) - \tan(\delta_2 - \beta)] \tag{12}$$

$$z_0 = z_1 + (x_0 - x_1)\tan(\delta_1 - \alpha) \tag{13}$$

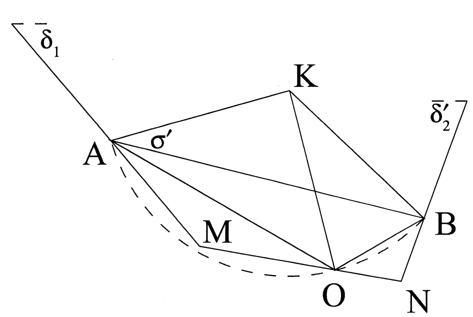

Fig. 4. Proof of the locus of the point of common tangency.

Eq. (13), which defines O with respect to A, is matched by:

$$z_0 = z_2 + (x_0 - x_2)\tan(\delta_2 - \beta), \tag{14}$$

which defines O with respect to B. Eq. (12) eliminates z_0 between Eq. (13) and Eq. (14).) The variable β may be eliminated from Eq. (12) by using Eq. (3), to give an equation dependent on α only.

For further computation it is convenient to redefine the problem in an axis frame (x',z') with origin at O and in which the common tangent MON is horizontal (i.e. the axis frame is rotated through $(\delta_1 - 2\alpha)$: see Fig. 3). The normal at O then becomes the z' axis, and the x' co-ordinates of the arc centres, P and Q, are both zero. The z' co-ordinate of P (which also defines the radius AP) is:

$$z'_P = (x'^2_1 + z'^2_1)/(2z'_1). \tag{15}$$

(This can be established by applying the sine rule to the triangle AOP.) A similar relationship, in terms of x'_2, z'_2, defines z'_Q for Q.

The centre of the circular arc which is the locus of O, the point of common tangency, is:

$$z'_K = [x'_2 - x'_1 - z'_2 \cot 2\beta + z'_1 \cot 2\alpha]/(\tan\beta - \tan\alpha); \tag{16}$$

$$x'_K = x'_1 - z'_1 \cot 2\alpha - z'_K \tan\alpha. \tag{17}$$

[Eq. (17) expresses the fact (see Fig. 4) that K lies on a line through M (for which $x'_M = x'_1 - z'_1 \cot 2\alpha$) at an angle of $-(90 - \alpha)$ to MON, i.e. the x' axis. A similar equation follows since K lies on a line through N at an angle of 2β. Eq. (16) arises by eliminating x'_K between the two equations.] Again, Eq. (3) may be used to eliminate β from Eq. (16). The radius of the circular arc is clearly:

$$r_K = [(x'_1 - x'_K)^2 + (z'_1 - z'_K)^2]^{1/2}. \tag{18}$$

The limiting values of α, which define the range of possible solutions, are:

$$\alpha = \delta_1 - \gamma,\ (\delta_1 + \delta_2)/2 - \gamma, \tag{19}$$

where γ is the angle of dip of the line AB. (The first limit corresponds to the line tangential to the bedding at A; the second to the line tangential to the bedding at B, so that $\beta = \delta_2 - \gamma$, which is recast in terms of α using Eq. (3).

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